M.B. PATEL SCIENCE COLLEGE, ANAND.

T.Y. B.Sc Internal Exam.

Monday, 17th October - 2011

MATHEMATICS : M - 301

(REAL ANALYSIS)

Time: 3:00 to 4:30 p.m. MARKS: 50

Q:1 A] Attempt any three of the following. (15)

1) Show that there is no rational number whose square is 7.

2) Prove that there exists a positive number π, such that $c(\pi/2)=0$ and $c(x)>0$ for $0\leq x\leq \pi/2$, Hence prove that $s(\pi/2)=1$.

3) for all real numbers x and y show that

(a) $|x-y|\geq ||x|-|y||$

(b) $\left|\frac{x}{y}\right|=\frac{|x|}{|y|}$, where $y\neq 0$.

4) If a and b be any two positive real numbers then show that there exists a positive integer n such that $na>b$.

5) Prove that $E(x)=e^{x}$ for all $x\in R$.

B] Attempt any one of the following. (02)

1) In usual notation prove that $L(ab)=L(a)+L(b)$

2) for all real number x and y show that $|x+y|\leq |x|+|y|$.

Q:2 A] Attempt any three of the following. (15)

1) Prove that the set of limit points of a bounded sequence has the greatest and the least member.

that every convergent seq^n is bounded and has a unique limit.
3) State and prove Bolzano-Weierstrass theorem for sequence.
4) ① Define limit point of a sequence, and show that every bounded sequence with unique limit point is convergent.
5) show that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

3] Attempt any one of the following. (0
1) find upper limit and lower limit of the seq^n : $S_n = \frac{(-1)^{n-1}}{n!}$, $\forall n \in N$.
2) show that $(-1)^n$ has two limit points.

Q:3 A] Attempt any three of the following (15)
1) Show that the interior of a set contains every open subset of a set.
2) Define neighbourhood of a point and show that intersection of two neighbourhood is also neighbourhood.
3) In usual notations prove that $(S \cup T)' = S' \cup T'$
4) State and prove Bolzano-Weierstrass theorem for a set.
5) Show that every open set is a union of open interval.

B] Attempt any one of the following. (01)
1) Define : Derived set.
2) Find the derived set of the set,
$A = \{x \in R \mid 0 < x < 1\}$

ALL THE BEST

M. B. Patel Science College, Anand

1st Internal Examination, October-2011 Date: 18/10/11

T. Y. B.Sc., Mathematics M 302 Marks: 50

Q.1

(a) Prove that a connected graph is an Euler graph iff all vertices of G have even degree. [5]

(b) If G has n-vertices and k-components then prove that G can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges. [5]

(c) Define: Euler line, Parallel Edges, Adjacent Edges, Isomorphic Graphs, Walk. [5]

Q.2

(a) For a given sequence of integers 4, 1, 3, 7, 0, 2, 8 find the largest monotonically increasing subsequence of it by using graph theory. [5]

(b) Prove that a tree has either one or two centres. [5]

(c) Define: Ecentricity of a vertex, Rooted Tree, Fundamental Circuit, Level of a vertex, Binary Tree. [5]

Q.3

(a) Describe operation - 1 [5]

(b) Prove that two 2-isomorphic graphs have circuit correspondence. [5]

(3) State and prove Euler's Theorem. [5]

(4) Prove that K_5 has no dual. [5]

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M. B. PATEL SCIENCE COLLEGE, ANAND.
First Internal examination
2011.
T. Y. B. Sc.

DATE: 19/10/2011.

Mathematics
M: 303
(Topology).

50 Marks

Q:1 Attempt any three (17).

(a): Define: Countable set.
Prove that countable union of countable set is countable [6]

(b): State and Prove Leibnitz's Theorem for convergence of alternating series of real numbers [6]

(c): When a series of real number is convergence
1. By using your defination Prove that [5]
$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converes to 1

(d): Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{x^{n-1} \cdot n^n}{n!}$ is convergent [5]

(e): Define: Absolutely convergence of a series. P.T every absolutely convergence of a series of real number is convergent. [5]

Q:2 Attempt any three (17)

(a): State and Prove Cauchy-criteria for uniform convergence of a sequence of functions. [5]

(b) State and Prove 'Dini's' Theorem for uniform convergence of a sequence of functions [6]

(c) If g is continuous function on a closed and bounded interval $[a,b]$. & $\{f_n\}$ is a sequence of continuous functions which converges to f on $[a,b]$ then Prove that $\lim_{n\to\infty} \int_a^b f_n g = \int_a^b f \cdot g$. [6]

P.T.O For Page 2

Q.2 (d) $f_n(x) = \frac{nx}{1+n^2x^2}$, $-\infty < x < \infty$. [5]

Does $\{f_n\}_1^\infty$ converges uniformly on $(-\infty, \infty)$? Justify your answer.

(e) $\{f_n\}_{n=1}^\infty$ is a sequence of function in $R[a,b]$ which converges uniformly to f on $[a,b]$ then Prove that $f \in R[a,b]$ & $\lim_{n\to\infty} \int_a^b f_n\,dx = \int_a^b (\lim_{n\to\infty} f_n)\,dx$ [5]

Q.3 Attempt any three.

(a) Define: metric and matric space. [6]

$\rho: R^2 \times R^2 \to [0,\infty)$ defined by $\rho(x,y) = [(x_1-x_2)^2 + (y_1-y_2)^2]^{1/2}$ $\forall\, x = (x_1,y_1),\ y = (x_2,y_2) \in R^2$. P.T $\langle R^2, \rho \rangle$ is a metric space.

(b) When two metrices ρ and σ are said to be equivalent? $P, Q \in R^2$, $P(x_1,y_1)$, $Q(x_2,y_2)$ [6]

$\rho(P,Q) = |x_1-x_2| + |y_1-y_2|$.

$\sigma(P,Q) = \max\{|x_1-x_2|, |y_1-y_2|\}$. P T or dis.

Prove that σ and ρ are equivalent.

(c): $f: M_1 \to M_2$. defined by $f(x) = 2x\ \forall x \in M_1$ where $M_1 = [0,1]$, $M_2 = [0,2]$ with absolute value metric. [5]

(1) Does f a continuous function on M_1?

(2) Does f^{-1} a continuous function on M_2?

(3) Does f a Homeomorphism?

(d): If $\mathcal{F}$ is a family of closed subset of a metric space M. Prove that $\bigcap_{F\in\mathcal{F}} F$ is closed set in M. What you say about $\bigcup_{F\in\mathcal{F}} F$? [5]

(e) Define: closure of a set.
closed subset. [5]

If $E \subset M$, M is a metric space then Prove that $\bar{E}$ is closed subset of M.

Best- Luck

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M.B. PATEL SCIENCE COLLEGE,

T.Y. B.Sc. Internal examination

20/10/2011 , Thursday

MATHEMATICS - M: 304

(ABSTRACT ALGEBRA)

MARKS : 50

Q-1 (a): Let H & K be subgroups of group G. Then prove that HK is subgroup of G iff HK = KH. [5]

(b) Let H & K be finite subgroups of group G such that HK is a subgroup of G. Then prove that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$ [6]

(C) Prove that any subgroup of an infinite cyclic group is also an infinite cyclic group. [6]

OR

Q-1 (a) State and prove left cancelation law for group. [5]

(b) Prove that every subgroup of a cyclic group is also cyclic. [6]

(c) Prove that intersection of 2 subgroups of a group G is also a subgroup of G. [6]

Q-2 (a) State and prove Cayley's theorem. [5]

(b) State and prove First isomorphism theorem [6]

(c) Prove that a subgroup H is normal in group G iff every left coset of H in G is right coset of H in G. [6]

OR

Q-2 (a) Prove that any infinite cyclic group is isomorphic to Z. also prove any finite cyclic group of order n is isomorphic to Z_n. [6]

(b) Let $G = (a)$ be a finite cyclic group of order [6] n. Then Prove that the mapping $\theta : G \to G$ defind by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n

(c) Prove that G is a direct product of sub-groups H and K iff every $x \in G$ can be uniquely expressed as $x = hk$, $h \in H$, $k \in K$. [5]

Q:3

(a) Prove that every group of prime order is cyclic. [5]

(b) State and Prove lagrange's theorem. [5]

(c) Prove that every cyclic group is abelian. [6]

OR

Q:3

(a) State and Prove third isomorphism theorem [5]

(b) Let G be an infinite cyclic group. Then Prove that G has only one non-trivial automorphism. [5]

(c) Let $\theta : G \to G'$ be an homomorphism of G [6] onto G'. Let e & e' be the unit elements of G & G' respectively Then prove that (i) $\theta(e) = e'$ (ii) $\theta(a^{-1}) = \{\theta(a)\}^{-1}$, for all $a \in G$.

M. B. PATEL SCIENCE COLLEGE

INTERNAL TEST

T.Y.B.SC M-305

NUMBER THEORY & COMPLEX ANALYSIS

DATE: 21/10/11 MARK: 50 TIME: 3:00 to 4:30

Q:1 (a) Attempt any three. [15]

1) State and Prove fundamental theorem of divisibility.

2) If P_n is n^{th} prime number then Prove that $P_n < 2^{2^n}$, for all $n \in N$.

3) Find $(136, 221, 391)$ and $[136, 221, 391]$.

4) Prove that there are infinitely many prime number of the form $4n-1$.

5) Let g be a positive integer greater than 1 then Prove that every positive integer a can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + \dots + c_1 g + c_0$, where $n \geq 0$, $c_i \in \mathbb{Z}$, $0 \leq c_i < 1$, $c_n \neq 0$.

(b) Attempt any one. [2]

1) Prove that $(a-s) \mid (ab+st) \Rightarrow (a-s) \mid (at+sb)$.

2) Prove that $(a+b)[a,b] = b[a, a+b]$, $\forall a,b > 0$.

Q:2 (a) Attempt any three [15]

1) Prove that Euler's function is multiplicative function.

2) Define Fermat's number. Prove that every prime factor of F_n $(n \geq 2)$ is of the form $2^{n+2}t + 1$, for some integer t.

3) Prove that the system of congruences, $x \equiv a \pmod{m}$; $x \equiv a \pmod{n}$ has solution iff $a \equiv b \pmod{(m,n)}$. Also Prove that system has unique solution with respect to modulo $[m,n]$.

4) Solve the equation.

(i) $12x + 15 \equiv 0 \pmod{45}$

(ii) $18x \equiv 30 \pmod{42}$.

5) State and Prove chinese remainder theorem.

(b) Attempt any one [02]

1) State and prove Fermat's thm.

2) Prove that a set of k integers $a_1, a_2, a_3 \dots a_k$ is a reduced residue system modulo m iff (i) $k = \phi(m)$ (ii) $(a_i, m) = 1, \forall i$ (iii) $a_i \not\equiv a_j \pmod{m}$, $\forall i \neq j$.

Q:3. Attempt any three. [16]

1) State and prove chain rule for differentiating composite functions.

2) State and prove cauchy-riemann Equation.

3) Determine where $f'(z)$ exist for $f(z) = |z|^2$ by using C-R equations.

4) If $\lim_{z \to z_0} f(z) = w_0$, $\lim_{z \to z_0} g(z) = w_1$, then prove that:

(i) $\lim_{z \to z_0} [f(z) g(z)] = w_0 w_1$ (ii) $\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{w_0}{w_1}$ if $w_0 \neq 0$.

5) Prove that $f'(z)$ does not exist at any point for following $f(z)$:

(i) $f(z) = 2x + ixy^2$

(ii) $f(z) = e^{\bar{z}}$.

M. B. Patel Science College, Anand

1st Internal Examination, October - 2011 Date: 22/10/11

T. Y. B.Sc. Mathematics M 306 Marks: 50

Q.1

(a) Show that the component of gradient of a scalar field V in any direction is the rate of change of V in that direction, where $V = V(x, y, z)$ [5]

(b) If a point 'O' is circum-centre of ΔABC, and the forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ are acting along $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ respectively and are in equilibrium, then show that, [5]

$$\frac{P}{a^2(b^2+c^2-a^2)} = \frac{Q}{b^2(c^2+a^2-b^2)} = \frac{R}{c^2(a^2+b^2-c^2)}$$

(c) The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled, $\vec{R}$ is doubled and if $\vec{Q}$ is reversed, $\vec{R}$ is again doubled. Then show that. [5]

$$P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$$

Q.2

(a) A light rigid rod of length $2b$ terminated heavy particles of weight w and W is placed inside the smooth hemi-spherical bowl of radius 'a' which is fixed with its own rim horizontal. If the particle of weight w is rest just below the rim then prove that $wa^2 = W(2b^2 - a^2)$. [5]

(b) Show that there exists a mass centre of a system of particles and it is unique. [5]

(c) Show that the potential inside a thin spherical shell is constant. [5]

(P.T.O).

Q.3

(a) For a common catenary, prove that $y = c(\cosh(x/c) - 1)$ $\boxed{5}$

(b) A cable 200 feet long hangs between two points at the same height and depth is 20 feet to 'O'. The tension at either point of suspension is 120 lbwt. Find the total weight of the cable. $\boxed{5}$

(c) Prove that the rate of change of linear momentum of a system of particles is equal to the vector sum of all external forces. $\boxed{5}$

(d) Obtain the equations of motion of a projectile with resistance. $\boxed{5}$

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